Intermittency of Magnetohydrodynamic Turbulence: Astrophysical Perspective

A. Lazarian
University of Wisconsin-Madison, Dept. of Astronomy, 475 Charter St., Madison, WI 53706

Abstract. Intermittency is an essential property of astrophysical fluids, which demonstrate an extended inertial range. As intermittency violates self-similarity of motions, it gets impossible to naively extrapolate the properties of fluid obtained computationally with relatively low resolution to the actual astrophysical situations. In terms of Astrophysics, intermittency affects turbulent heating, momentum transfer, interaction with cosmic rays, radio waves and many more essential processes. Because of its significance, studies of intermittency call for coordinated efforts from both theorists and observers. In terms of theoretical understanding we are still just scratching a surface of a very rich subject. We have some theoretically well justified models that are poorly supported by experiments, we also have She-Leveque model, which could be vulnerable on theoretical grounds, but, nevertheless, is well supported by experimental and laboratory data. I briefly discuss a rather mysterious property of turbulence called "extended self-similarity" and the possibilities that it opens for the intermittency research. Then I analyze simulations of MHD intermittency performed by different groups and show that their results do not contradict to each other. Finally, I discuss the intermittency of density, intermittency of turbulence in the viscosity-dominated regime as well as the intermittency of polarization of Alfvenic modes. The latter provides an attractive solution to account for a slower cascading rate that is observed in some of the numerical experiments. I conclude by claiming that a substantial progress in the field may be achieved by studies of the turbulence intermittency via observations.

Keywords: turbulence, molecular clouds, MHD

1. What are Intermittency and Extended Self-Similarity?

Astrophysical fluids are, as a rule, turbulent and magnetized. These two properties are closely interrelated. Turbulence itself can amplify magnetic fields stretching and winding of magnetic field lines (see Batchelor 1950) and it is an important ingredient of a more regular astrophysical dynamo (see Mofatt 1978) that is generally believed to be the origin of magnetic fields on the scale from stars and accretion disks to galaxies (see a discussion of the problems of the traditional approach in a review by Vishniac, Lazarian & Cho 2003 and references therein). Magnetic fields, in their turn, constrain motions of ions, which decreases the diffusivity and increases the Reynolds number Re of the flow. The latter is the ratio of the eddy turnover time of a parcel of gas to the time required for viscous forces to show it appreciably. Mathematically $Re = LV_L/\nu$, where L and V_L are the scale of the flow and its velocity, respectively, while ν is the viscous diffusivity. Numerically, for most astrophysical astrophysical flows the Reynolds number is huge, e.g. $Re > 10^8$. Its magnetic counterpart that characterizes to what extend the magnetic field is frozen in the fluid may be even larger, e.g. $Rm > 10^{16}$. Such tremendous Re and Rm are not feasible to reproduce in numerical simulations neither now nor in at any foreseeable future[1]. Is it possible to understand MHD turbulence in these circumstances?

A lot of understanding of the turbulence has been achieved by studies of turbulence self-similarity. This property, which is also called scale-invariance, implies that fluid turbulence can be reproduced by the magnification of some part of it. Take the famous model of incompressible Kolmogorov turbulence as an example. In this model the energy is injected at a large scale L and forms eddies that transfer the energy to smaller and smaller scales. At the scales where the corresponding Reynolds number $Re_l \sim lv_l/\nu$ is much larger than unity, the dissipation over eddy turnover time $t_l \sim l/v_l$ is negligible. As the result, the energy cascades to smaller and smaller scales without much dissipation, i.e. $v_l^2/t_l \sim const$, which gives the well-known Kolmogorov power-law scaling for the eddies of scale l, namely, $v_l \sim l^{1/3}$. The cascade terminates at the dissipation scale, which provides an *inertial range* from L to $LRe^{-3/4}$, where Re is the Reynolds number corresponding to the flow at the injection scale L.

At the dissipation scales the self-similarity is known to fail with turbulence forming non-Gaussian dissipation structures as exemplified, e.g. in Biskamp (2003). Interestingly enough, present-day research shows that self-similarity is not exactly true even along the inertial range. Instead the fluctuations tend to get increasingly sparse in time and space at smaller scales. This property is called *intermittency*. Note, that the power-law scaling does not guarantee the scale-invariance or absence of intermittency.

From the practical point of view, self-similarity of turbulence simplifies our description of turbulent phenomena, while intermittency spoils the nice picture. If intermittency is Re or Rm number dependent phenomenon one cannot believe to the results of numerical simulations because of the aforementioned disparity in the numbers. In other words, it invalidates naive extrapolations of the dynamics of the turbulent fluid obtained to with computers to real astrophysical systems. In terms

[1] Note, that the currently available 3D simulations for 1024 cubes have Re and Rm limited by numerical diffusion of the order of only 10^4 and those numbers scale linearly with the cube size.

of astrophysical implications, intermittency can substantially change heating of interstellar gas. According to Falgarone et al. (2005, Falgarone 1999 and references therein) many chemical endothermal reactions (e.g. formation of CH^+) take place in intensive vortices within interstellar medium. Cho & Lazarian (2003) speculated that the mysterious structures in ISM on the AU spatial scale (see Marscher et al. 1993, Heiles 1997) can be explained by intermittency of MHD turbulence in the viscosity-damped regime. Particle diffusion and acceleration may be very different[2] in intermittent systems (see Honda & Honda 2005). These and many other issues can be settled when we know more about intermittency of MHD turbulence.

One way to do such studies is to investigate the scaling powers of longitudinal velocity fluctuations, i.e. $(\delta V)^p$, where $\delta V \equiv (\mathbf{V}(\mathbf{x}+\mathbf{r}) - \mathbf{V}(\mathbf{x}))\mathbf{r}/r$. The infinite set of various powers of $S^p \equiv \langle(\delta V)^p\rangle$, where $\langle..\rangle$ denote ensemble[3] averaging, is equivalent to the p.d.f. of the velocity increments. For those powers one can write $S^p(r) = a_p r_p^{\xi}$ to fully characterize the isotropic turbulent field in the inertial range. While the scaling coefficients a_p are given by the values of the function S^p e.g. at the injection scale, the scaling exponents ξ_p are very non-trivial. It is possible to show that for a self-similar flow the scaling exponents are linear function of n, i.e. $\xi_p \sim p$, which for Kolmogorov model $S^1 \sim v_l \sim l^{1/3}$ gives $\xi_p = p/3$. Experimental studies, however, give different results which shows that the Kolmogorov model is an oversimplified one.

MHD turbulence, unlike hydro turbulence, deals not only with velocity fluctuations, but also with the magnetic ones. The intermittencies of the two fields can be different. In addition, MHD turbulence is anisotropic as magnetic field affects motions parallel to the local direction of $\mathbf{B}$ very different. This all makes it more challenging to understand the properties of MHD intermittency more interesting.

In spite of the aforementioned limitations of the computational approach to studies of astrophysical turbulence, computers provide the most cost-effective way of *testing* turbulence scalings. The problem with both computational and laboratory studies of S^n is a relatively small inertial range.

An interesting and yet not understood property of structure functions, however, helps to extend the range over which S^p can be studied.

[2] Recent changes in understanding of MHD turbulence have induced substantial shifts in understanding of cosmic ray diffusion and acceleration (see Yan & Lazarian (2004), Cho & Lazarian (2005) and references therein). It may happen that intermittency is another vital missing ingredient of the modern cosmic ray propagation models.

[3] In astrophysics spatial or temporal averaging is used.

Benzi et al. (1993) reported that for hydrodynamic turbulence the functions $S^p(S^3)$ exhibit much broader power-law range compared to $S^p(r)$. While for the inertial range a similarity in scaling of the two functions stem from the Kolmogorov scaling $S^3 \sim r$, the power-law scaling of $S^p(S^3)$ protrudes well beyond the inertial range into the dissipation range[4] . This observation shows that the dissipation "spoils" different orders of S in the same manner. Therefore there is no particular need to use the third moment, but one can use any other moment $S^m \sim r^m$ and obtain a good power law of the function $S^p \sim (S^m)^{\xi_p/\xi_m}$ (see Biskamp 2003).

Even not understood, the above property of S^n allows to get insight into the scaling of realistic high Re flows with limited Re number simulations or laboratory experiments. This way the work of Benzi et al. (1993) allowed much of the progress that we describe below.

2. What are the measures of intermittency of MHD turbulence?

Studies of intermittency in incompressible hydro can be traced back to works of Obukhov (1962) and Kolmogorov (1962). According to the *refined similarity hypothesis* the fluctuations at a scale l can be presented as $\delta V_l \sim \epsilon_l l^{1/3}$, where ϵ_l is the energy dissipation rate averaged over l^3 volume. This model allows for intermittency and provides $\zeta_p = 1/3 + \mu_{p/3}$. The corresponding value of $\mu_p/3$ according to Yaglom (1966) is $\mu_{p/3} = \mu p(p-3)/18$, where $\mu = 2 - \xi_6$ and according to numerical simulations in Vincent & Meneguzzi (1991) is ~ 0.2. With this distribution it is possible to show that for a typical ISM injection scale of 50pc, 10% of the energy of the most energetic events, deposited within a typical dissipation scale of Alfvenic motions of several hundred of km (the Larmor radius of a proton in the cool neutral gas) is localized in just 10^{-2}% of the volume!

However, a more successful model to reproduce both experimental hydro data and numerical simulations is She-Leveque (1994) model. According to Dubrulle (1994) this model can be derived assuming that the energy from large scale is being transferred to $f < 1$ less intensive eddies and $1 - f$ of more intensive ones. The scaling relations suggested by She & Leveque (1994) related ζ_p to the scaling of the velocity $V_l \sim l^{1/g}$, the energy cascade rate $t_l^{-1} \sim l^{-x}$, and the co-dimension of the

[4] In practical terms this means that instead of obtaining S^p as a function of r, one gets S^p as a function of S^3, which is nonlinear in a way to correct for the distortions of S^p.

dissipative structures C:

$$\zeta_p = \frac{p}{g}(1-x) + C\left(1-(1-x/C)^{p/g}\right). \qquad (1)$$

For incompressible turbulence these parameters are $g=3$, $x=2/3$, and $C=2$, implying that dissipation happens over 1D structures (e.g. vortices). There are theoretical arguments against the model (see Novikov 1994), but so far the She-Leveque scaling is the best for reproducing the intermittency of incompressible hydrodynamic turbulence.

In their pioneering study Muller & Biskamp (2000) applied the She-Leveque model to incompressible MHD turbulence and attracted the attention of the MHD researchers to this tool. They used Elsasser variables and claimed that their results are consistent with dissipation within 2D structures (e.g. 2D current sheets). The consequent study by Cho, Lazarian & Vishniac (2002a) used velocities instead of Elsasser variables and provided a different answer, namely, that the dimension of dissipation structures is the same as in incompressible hydro, i.e. the dissipation structures are 1D. The difference between the two results was explained in Cho, Lazarian & Vishniac (2003, henceforth CLV03). They noted that, first of all, the measurements in Muller & Biskamp (2000) were done in the reference frame related to the *mean* magnetic field, while the measurements in Cho, Lazarian & Vishniac (2002a) were done in the frame related to the *local* magnetic field. We believe that the latter is more physically motivated frame, as it is the local magnetic field is the field that is felt by the eddies. It is also in this reference frame that the scale-dependent anisotropy predicted in the Goldreich-Shridhar (1995, henceforth GS95) model is seen. Computations in CLV03 confirmed that the dissipation structures that can be identified as velocity vortices in the local magnetic field reference frame can also be identified with two dimensional sheets in terms of Elsasser variables in the mean magnetic field reference frame. This, first of all, confirms a mental picture where motions perpendicular to magnetic field lines are similar to hydrodynamic eddies. More importantly, it sends a warning message about the naive interpretation of the She-Leveque scalings in the MHD turbulence.

Intermittency in compressible MHD turbulence was discussed in Boldyrev (2002) who assumed that the dissipation there happens in shocks and therefore the dimension of the dissipation structures is 2. The idea of the dominance of shock dissipation does not agree well with the numerical simulations in Cho & Lazarian (2002, 2003, henceforth CL03), where the dominance of the vortical motions in *subAlfvenic* turbulence (i.e. magnetic pressure is larger than the gaseous one) was reported. Nevertheless, numerical simulations in Padoan et al (2003)

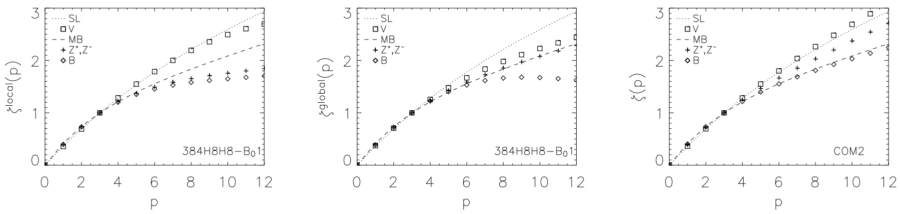

Figure 1. *left panel*: Intermittency exponents for incompressible MHD turbulence in perpendicular directions in the local frame. The velocity exponents show a scaling similar to the She-Leveque model. The magnetic field shows a different scaling. *central panel*: Intermittency exponents for incompressible MHD turbulence in the global frame. Note that the result for $z^{\pm}$ is very similar to the Müller-Biskamp model. *right panel*: Intermittency exponents for superAlfvenic compressible turbulence in the global frame. From CLV03

showed that for *superAlfvenic* turbulence (i.e. magnetic pressure is less than the gaseous one) the dimension of the dissipation structures was gradually changing from one to somewhat higher than two as the Mach number was increasing from 0.4 to 9.5. The very fact that the super-Alfvenic turbulence, which for most of the inertial scale resolvable by simulations does not have a dynamically important magnetic field is different from subAlfvenic is not surprising. The difference between the results in Padoan et al. (2003) at low Mach number and the incompressible runs in Muller & Biskamp (2000) deserves a discussion, however. First of all, the results in Padoan et al. (2003) are obtained for the velocity, while the results in Muller & Biskamp (2000) are obtained for the Elsasser. CLV03 has shown that the magnetic field and velocity have different intermittencies. Indeed, it is clear from Fig. 1 that $\zeta^{\mathrm{magnetic}} < \zeta^{\mathrm{velocity}}$ which means that magnetic field is more intermittent than velocity. An interesting feature of superAlfvenic simulations in Fig. 1 is that the velocity follows the She-Leveque (1994) hydro scaling with vortical dissipation, while magnetic field exhibits a pronounced dissipation in current sheets. Both features are expected if magnetic field is not dynamically important and the turbulence stays essentially hydrodynamic. We also see that the dynamically important magnetic field does changes the intermittency. The flattening of magnetic field scaling is pronounced in Fig. 1.

3. What are the intermittencies of Alfven, slow and fast components of MHD cascade?

A further studies of intermittencies was done in Kowal & Lazarian (2006). There we used a decomposition of the MHD turbulence into Alfven, slow and fast modes from Cho & Lazarian (2002)(see Fig. 2).

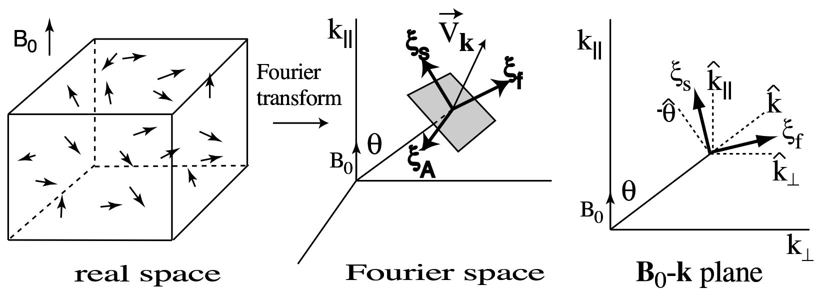

Figure 2. Separation method. We separate Alfven, slow, and fast modes in Fourier space by projecting the velocity Fourier component $\mathbf{v_k}$ onto bases ξ_A, ξ_s, and ξ_f, respectively. Note that $\xi_A = -\hat{\varphi}$. Slow basis ξ_s and fast basis ξ_f lie in the plane defined by $\mathbf{B}_0$ and $\mathbf{k}$. Slow basis ξ_s lies between $-\hat{\theta}$ and $\hat{\mathbf{k}}_\parallel$. Fast basis ξ_f lies between $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}_\perp$. From CL03

How physical is this decomposition? If the coupling between the modes is strong in MHD turbulence one cannot talk about three different energy cascades. Indeed, the compressible MHD turbulence is a highly non-linear phenomenon and it has been thought that Alfven, slow and fast modes are strongly coupled. Nevertheless, one may question whether this is true. A remarkable feature of the GS95 model is that Alfven perturbations cascade to small scales over just one wave period, while the other non-linear interactions require more time. Therefore one might expect that the non-linear interactions with other types of waves should affect Alfvenic cascade only marginally. Moreover, since the Alfven waves are incompressible, the properties of the corresponding cascade may not depend on the sonic Mach number.

The generation of compressible motions (i.e. *radial* components in Fourier space) from Alfvenic turbulence is a measure of mode coupling. How much energy in compressible motions is drained from Alfvenic cascade? According to closure calculations (Bertoglio, Bataille, & Marion 2001; see also Zank & Matthaeus 1993), the energy in compressible modes in *hydrodynamic* turbulence scales as $\sim M_s^2$ if $M_s < 1$. CL03 conjectured that this relation can be extended to MHD turbulence if, instead of M_s^2, we use $\sim (\delta V)_A^2/(a^2 + V_A^2)$. (Hereinafter, we define $V_A \equiv B_0/\sqrt{4\pi\rho}$, where B_0 is the mean magnetic field strength.) However, since the Alfven modes are anisotropic, this formula may require an additional factor. The compressible modes are generated inside the so-called GS95 cone, which takes up $\sim (\delta V)_A/V_A$ of the wave vector space. The ratio of compressible to Alfvenic energy inside this cone is the ratio given above. If the generated fast modes become isotropic (see below), the diffusion or, "isotropization" of the fast wave energy in the wave vector space increase their energy by a factor of $\sim V_A/(\delta V)_A$.

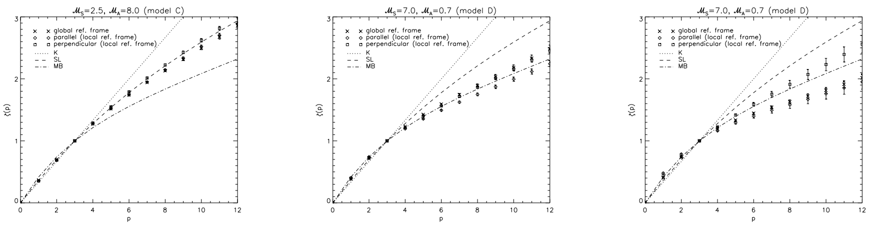

Figure 3. Intermittency of velocities. K-no intermittency, SL- She-Leveque model with dissipation in 1D structures, MB- She-Leveque model with dissipation in 2D structures. *left panel*: Intermittency exponents for Alfvenic modes. *central panel*: Intermittency exponents for slow modes. *right panel*: Intermittency exponents for fast modes. From Kowal & Lazarian 2006.

This results in

$$\frac{\delta E_{comp}}{\delta E_{Alf}} \approx \frac{\delta V_A V_A}{V_A^2 + c_s^2}, \qquad (2)$$

where δE_{comp} and δE_{Alf} are energy of compressible and Alfven modes, respectively. Eq. (2) suggests that the drain of energy from Alfvenic cascade is marginal when the amplitudes of perturbations are weak, i.e. $(\delta V)_A \ll V_A$. Results of numerical calculations shown in Cho & Lazarian (2002) support these theoretical considerations. This justifies[5] our treating modes separately.

Our study in Kowal & Lazarian (2006) showed that the intermittency of Alfven modes, as we expected, do not change appreciably with Mach number. The corresponding dimension of the dissipation structures is close to one, if the calculations are done in terms of velocities and local system of reference is used. The change in the intermittencies of slow and fast modes is more pronounced. The corresponding plots are given in Fig. 3.

4. What is the intermittency of density?

Density fluctuations depend on the Mach number of turbulence. For high Mach numbers the density spectrum gets shallow which is related to the clumps and density sheets created by shocks (see Padoan et al. 2004, Cho & Lazarian 2004). These shocks constitute a small fraction of volume, while in the bulk of the volume the shearing motions affect the density structure. In Beresnyak, Lazarian & Cho (2005) it was

[5] A claim in the literature is that a strong coupling of incompressible and compressible motions is required to explain simulations that show fast decay of MHD turbulence. There is not true. The incompressible motions decay themselves in just one Alfven crossing time.

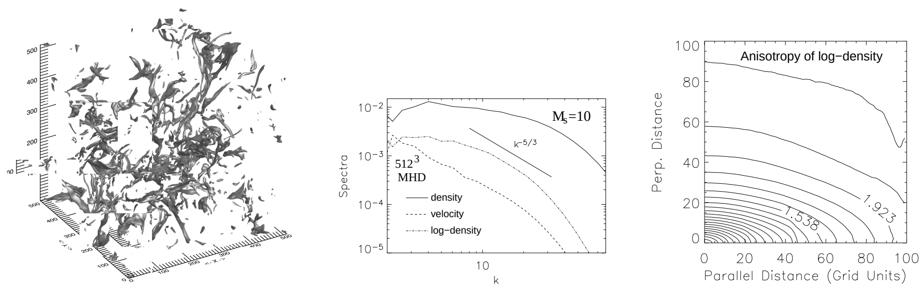

Figure 4. *Left*: The isosurfaces of density, corresponding to 10 mean densities. *Middle*: Spectra of log density tends to be Kolmogorov, while the density at high Mach number is shallow. *Right* The contours of isocorrelation of $\log\rho$ are very similar to those for velocity in GS95 picture. By Beresnyak & Lazarian.

shown that the logarithm of density exhibits both GS95 spectrum and anisotropy[6] (see Fig. 4). Our intermittency study in Kowal & Lazarian (2006) showed that the fluctuations of $\log\rho$ are much more regular than the fluctuations of ρ. The intermittency of $\log\rho$ is somewhat higher than that of velocity (Fig. 5). Nevertheless, the remarkable regularity of $\log\rho$ should have its physical explanation. A possible one is related to the multiplicative symmetry with respect to density in the equations for isothermal hydro (see Passot & Vazquez-Semadeni 1998). This means that if a stochastic process disturbs the density, it results in perturbations of density being multiplied rather than added together. Consequently, the distribution of density for a Gaussian driving of turbulence tends to be lognormal, which is consistent with the probability distribution function measurements in Beresnyak, Lazarian & Cho (2005). In Kowal & Lazarian (2006) we used a *genus* measure (see Gott et al. 1990) and confirmed that $\log\rho$ is close to Gaussian in our MHD simulations.

Magnetic forces should affect the multiplicative symmetry above. However, they do not affect the compressions of gas parallel to magnetic field. Those compressions in magnetically dominated fluid will be of highest intensity and therefore most important. They are sheared by Alfvenic modes, as their own evolution will be slower than that imposed by the Alfvenic cascade (see theoretical considerations in GS95, CL03). We note that the shearing does not affect the pdfs, but it does affects the spectra and anisotropy of the turbulence. This explains close relationship between the properties of velocity in our simulations and those of density.

[6] We note that the density itself does not show anisotropy, which means that the large density peaks that scale with the squared of the Mach number, are not much affected by magnetic field.

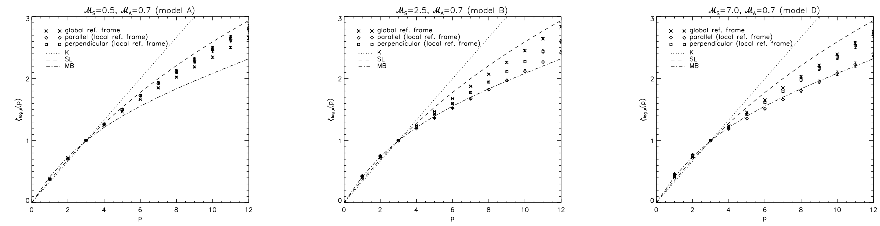

Figure 5. Intermittency of density for different models computed. K-no intermittency, SL- She-Leveque model with dissipation in 1D structures, MB- She-Leveque model with dissipation in 2D structures. From Kowal & Lazarian 2006.

The advantages of using $\log \rho$ rather than ρ for studies of supersonic turbulence were first realized in Porter et al. (1998ab). However, important attempts to study the intermittency of density rather than its logarithm were made in Boldyrev et al. (2002) and Padoan et al. (2003).

5. What is the intermittency of MHD turbulence in the viscosity-damped regime?

Viscosity may be higher than resistivity, for instance, in a partially ionized gas. The neutrals do not follow magnetic field lines and thus produce viscosity, while their effect to conductivity is less pronounced.

In hydro turbulence viscosity sets a viscous scale with motions on smaller scales being exponentially suppressed. This is natural, as at the viscous scale the kinetic energy is dissipated rather than being transferred further. This means the end of hydro cascade, but the MHD turbulence does evolve below the viscous scale, provided that the resistivity is lower than viscosity. Indeed, magnetic fields can be stretched by turbulent motions at larger scales, with the shear from the eddies at the damping scale being most important. This means a new, viscosity-damped regime of MHD turbulence (Cho, Lazarian, & Vishniac 2002b).

A theoretical model for this regime of turbulence is given in Lazarian, Vishniac, & Cho (2004, henceforth LVC04). There it is shown that both velocities and densities below the viscous scale form at a scale l intermittent structures with the scale-dependent filling factor ϕ_l, $v_l^2 = \phi_l \hat{v_l}$, $b_l^2 = \phi_l \hat{b_l}$, where $\hat{v_l}$ and $\hat{b_l}$ correspond to, respectively, velocity and magnetic field within the subvolumes. The cascading happens on the turnover time for the eddies at the viscous scale, which means $b_l^2 \sim const$. For the incompressible fluid LVC04 predicts that the filling factor ϕ_l is proportional to l; the velocity spectrum *below* viscosity

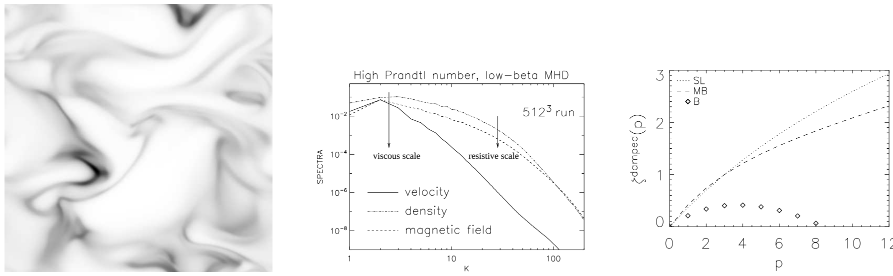

Figure 6. Left: Filaments of density created by magnetic compression in the slice of data cube of the viscosity-damped regime of MHD turbulence. *Middle*: Spectra of density and magnetic field are similar, while the velocity is damped (as a steep power law). Note that in this regime the resistive scale is $LRm^{-1/2}$. *Right*: Intermittency spectral index for the incompressible MHD turbulence in the viscosity-damped regime.

cutoff scales as k^{-4}, while the magnetic field spectrum scales as k^{-1}. This all is consistent with the incompressible numerical simulations in CLV03, but the theory may need to be modified for the compressible medium, where the magnetic filaments are allowed to expand.

For the extreme intermittency of the magnetic field suggested in LVC04 the higher moments of structure functions $S_p \sim \hat{b}_l^{\,p} \phi_l$ which means that $S_p \sim l^{1-p/2}$. The concentration of magnetic field in thin filaments gives rise to resistive loses that should eventually make $\xi_p = 0$ for sufficiently large p. In Fig. 6 we see this general tendency for high p. For the absence of the more precise correspondence we may blame (a) our crude model for estimating ξ, (b) numerical effects, and (c) LVC04 model itself. Addressing the issue (b), we would say that the compelling arguments in the model provide k^{-1} spectrum and this would provide $\xi(2) = 0$ in accordance with the intermittency model above. However, due to numerical effects identified in LVC04 the spectrum of magnetic fluctuations is slightly steeper.

6. What is the intermittency in polarization?

Polarization is a more subtle effect that is observed in Alfvenic turbulence. Since Alfven waves are transversal, i.e. the velocities are perpendicular to both $\mathbf{B}$ and $\mathbf{k}$, they have two polarizations. It can be shown that only waves with orthogonal polarizations efficiently interact with each other (see Biskamp 2003). Consider two oppositely moving wave packets of the *same* energy, partially polarized in the same direction. Their interaction can be roughly understood as the interaction of 2 pairs of orthogonally polarized wave packets (i.e. 4 wave packets total).

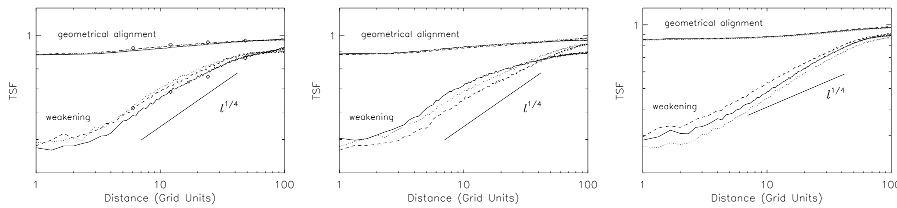

Figure 7. The geometrical alignment (upper curve) and the weakening of interaction factor (lower curve), *left panel*: incompressible, $M_A = 0.7$, Solid, dotted and dashed lines are for three simulations, separated by Alfvénic cross time. *Central panel*: incompressible, $M_A = 1.0$, same notation. *Right panel*: Compressible, $M_A = 1$, $M_s = 1$, we used a global mean magnetic field for both mode separation and scale separation. Clearly in all these cases pure geometric alignment provides rather marginal effect. The weakening of the interaction arises from intermittent regions with high polarization and high wave amplitudes.

Within each of these packets there is *imbalance*, i.e. the energy in one of the interacting waves is larger than the energy of the other wave moving in the opposite direction. Studies of the imbalanced turbulence (see Biskamp 2003, Cho, Lazarian & Vishniac 2002, Boldyrev 2005 and ref. therein) show that the stronger component induces a faster decay of the weaker one (it shears it more). As the result, the polarizations of the wave packets increase and the rate of turbulence decay decreases. Note, that the increase of degree of polarization was observed in the simulations by Maron & Goldreich (2001, henceforth MG01).

The recent upsurge of interest to effects of polarization is related to the ongoing attempts to understand the actual slope of the MHD turbulence spectrum. As the numerical simulations obtained a sufficient inertial range, it became possible to distinguish the spectral slopes $-5/3$ and $-3/2$. In particular, Boldyrev (2005, 2006) associated the $-3/2$ slope measured in MHD simulations of low Alfven Mach number ($M_A \ll 1$) turbulence (MG01, Muller & Grappin 2005) with the additional asymmetries of turbulent eddies in the plane perpendicular to magnetic field. This was in contrast to a remark in MG01 that related the flattening of the spectrum to the turbulence intermittency.

Our study in Beresnyak & Lazarian (2006) shows that the *polarization intermittency* plays an important role in MHD turbulence. We found that the MHD turbulence volume spontaneously develops regions that are characterized by both high amplitude of fluctuations and high degree of polarization (see Figure 7). At the same time the volume-averaged pure geometrical alignment of the oppositely moving wave packets is rather marginal for our simulations. The degree of the spontaneous polarization and the corresponding weakening of the cascade increases with the decrease of the scale. As the result one should expect the turbulence spectrum to get flatter.

An additional interesting effect noticed in Beresnyak & Lazarian (2006) is that the intermittency of the nonlinear term constructed with the fluctuations of the Elsasser variables u and w multiplied by $\sin\theta$, where θ is the angle between $\mathbf{w}$ and $\mathbf{u}$ is less intermittent than product u and w which can be interpreted as the tendency of turbulence to "self-organize" to provide a more homogeneous dissipation. Interestingly enough, the nonlinear term demonstrated much better extendended self-similarity than the u and w product. This indicates a rather fundamental nature of the nonlinear term. Naturally, more studies are required for this opening field.

7. What is the future of the field?

Extended self-similarity allows to perform informative intermittency studies using computers. A substantial impetus can be obtained from observational studies. Such studies have been performed already (see Falgarone et al. 2005, Padoan et al. 2003), but velocity statistics is still illusive. A substantial progress in understanding of when the observations reflect the statistics of velocity (see Esquivel & Lazarian 2005 and references therein) allows us to hope that measures of the intermittency of interstellar velocities will be available soon. This all should allow predictions of the intermittency theory to be tested with observations. A developed intermittency theory should initiate a chain reaction of changes within different branches of astrophysics, including the transport theory, acceleration of cosmic rays and, possibly, even astrochemistry. It also should contribute to a better interpretation of the simulation results in terms of astrophysical phenomena and may eventually results in new computational tools that would adequately represent the turbulence-related astrophysical processes at any scale of interest.

AcknowledgmentsI thank Ethan Vishniac, Jungyeon Cho and Andrey Beresnyak for valuable discussions of turbulence intermittency and effects of polarization on the Alfvenic cascade. The work is supported by the NSF Grant AST-0307869 and the NSF Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas.

References

Batchelor, G.K. 1950, Proc. Roy. Soc. London, A, 201, 405

Benzi, R., Ciliberto, S., Tripiccione, R., Boudet, C., Massaoili, F., & Succi, S. 1993, Phys. Rev. E, 48, R29

Beresnyak, A. & Lazarian, A. 2006, ApJ, 640, L175

Beresnyak, A., Lazarian, A. & Cho, J. 2005, ApJ, 624, L93

Bertoglio, J.-P, Bataille, F., & Marion, J.-D. 2001, Phys. Fluids, 13, 290

Biskamp, D. 2003, Magnetohydrodynamic Turbulence, CUP

Boldyrev, S. 2002, ApJ, 569, 841

Boldyrev, S. 2005, ApJ, 626, 37

Boldyrev, S. 2006, Phys. Rev. Lett, 96, Issue 11, 115002

Boldyrev, S., Norlund, A., & Podoan, P. 2003, Phys. Rev. Lett., 89, 031102

Cho, J., Lazarian, A. 2002a, Phys. Rev. Lett., 88, 245001 (CL02)

Cho, J., Lazarian, A. 2003, MNRAS, 345, 325

Cho, J., Lazarian, A. 2004, in Studying Turbulence Using Numerical Simulation, Center for Turbulence Research, Stanford, p. 75

Cho, J., Lazarian, A. 2006, ApJ, 638, 811

Cho, J., Lazarian, A., & Vishniac, E. 2002a, ApJ, 564, 291

Cho, J., Lazarian, A., & Vishniac, E. 2002b, ApJ, 566, L49

Cho, J., Lazarian, A., & Vishniac, E. 2003, ApJ, 595, 812

Dubrulle, B. 1994, Phys. Rev. Lett., 73, 959

Esquivel, A. & Lazarian, A. 2005, ApJ, 631, 320

Falgarone, E. 1999, in *Interstellar Turbulence*, ed. by J. Franco, A. Carraminana, CUP, p.132

Falgarone, E., Hily-Blant, P., Pety, J., Pineau Des Fors, G. 2005, in Magnetic Fields in the Universe, Eds. E. de Gouveia Dal Pino, G. Lugones, A. Lazarian, AIP, V. 784, 299

Heiles, C. 1997, ApJ, 481, 193

Honda, M. & Honda, Y.S. 2005, astro-ph/0510758

Goldreich, P. & Sridhar, S. 1995, ApJ, 438, 763

Gott, J.R., Park, C., Juskiewicz, R., Bies, W., Bouchet, F., Stebbins, A. 1990, ApJ, 352, 1

Kolmogorov, A.N. 1962, J. Fluid Mech. 13, 82

Kowal, G. & Lazarian, A. 2006, ApJ, submitted

Lazarian, A., Vishniac, E., & Cho, J. 2004, ApJ, 603, 180

Maron, J. & Goldreich, P. 2001, ApJ, 554, 1175

Mofatt, K. 1978, Magnetic Field Generation in Electrically Conducting Fluids, CUP

Marscher, A. P., Moore, E. M., & Bania, T. M. 1993, ApJ, 419, L101

Muller, W.,-C., & Biskamp, D. 2000, Phys. Rev. Lett. 84, 475

Muller, W., C. & Grappin, R. 2005, Phys. Rev. Letters, 95, Issue 11, 114502

Novikov, E., A. 1994, Phys. Rev. E, 50, R3303

Obukhov, A., M. 1962, J. Fluid Mech. 13, 77

Padoan, P., Boldyrev, S., Langer, W., & Norlund, A. 2003, ApJ, 583, 308

Padoan, P., Jimenez, R., Juvela, M., & Norlund, A. 2004, ApJ, 604, L49

Passot, T. & Vazquez-Semadeni, E. 1998, Phys. Rev. E, 58(4), 4501

Porter, D.H., Pouquet, A. & Woodward, P. 1998a, Phys. Fluids, 10, 237

Porter, D.H., Pouquet, A. & Woodward, P. 1998b, in Advances in Turbulence VII, ed. U. Frisch, Kluwer, p. 255

She, Z. & Leveque, E., 1994, Phys. Rev. Lett., 72, 336

Vincent, A. & Meneguzzi, M. 1991, J. Fluid Mech., 225, 1

Vishniac, E., Lazarian, A., & Cho, J. 2003a, in *Turbulence and Magnetic Fields in Astrophysics*, eds. E. Falgarone & T. Passot (Springer LNP), p. 376

Yaglom, A.M 1966, Dokl. Akad. Nauk SSSR, 166, 49

Yan, H. & Lazarian, A. 2004, ApJ, 614, 757
Zank, G.P. & Matthaeus, W.H. 1993, Phys. Fluids A., 5(1), 257